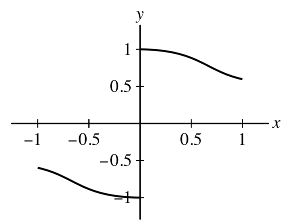
SOLUTION

(a)

x	-0.3	-0.2	-0.1	0.1	0.2	0.3
$f(x)$	-0.980506	-0.998049	-0.999998	0.999998	0.998049	0.980506

(b) As $x \to 0-$, $f(x) \to -1$, whereas as $x \to 0+$, $f(x) \to 1$.



2.3 Basic Limit Laws

Preliminary Questions

1. State the Sum Law and Quotient Law.

SOLUTION Suppose $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ both exist. The Sum Law states that

$$\lim_{x\to c}(f(x)+g(x)) = \lim_{x\to c} f(x) + \lim_{x\to c} g(x).$$

Provided $\lim_{x\to c} g(x) \neq 0$, the Quotient Law states that

$$\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}.$$

2. Which of the following is a verbal version of the Product Law (assuming the limits exist)?

(a) The product of two functions has a limit.

(b) The limit of the product is the product of the limits.

(c) The product of a limit is a product of functions.

(d) A limit produces a product of functions.

SOLUTION The verbal version of the Product Law is **(b)**: The limit of the product is the product of the limits.

3. Which statement is correct? The Quotient Law does not hold if:

(a) The limit of the denominator is zero.

(b) The limit of the numerator is zero.

SOLUTION Statement **(a)** is correct. The Quotient Law does not hold if the limit of the denominator is zero.

Exercises

In Exercises 1–24, evaluate the limit using the Basic Limit Laws and the limits $\lim_{x\to c} x^{p/q} = c^{p/q}$ *and* $\lim_{x\to c} k = k$.

1. $\lim_{x\to 9} x$

SOLUTION $\lim_{x\to 9} x = 9$.

2. $\lim_{x\to -3} 14$

SOLUTION $\lim_{x\to -3} 14 = 14$.

3. $\lim_{x\to \frac{1}{2}} x^4$

SOLUTION $\lim_{x\to \frac{1}{2}} x^4 = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$.

4. $\lim_{z\to 27} z^{2/3}$

SOLUTION $\lim_{z\to 27} z^{2/3} = 27^{2/3} = 9.$

5. $\lim_{t\to 2} t^{-1}$

SOLUTION $\lim_{t\to 2} t^{-1} = 2^{-1} = \frac{1}{2}.$

6. $\lim_{x\to 5} x^{-2}$

SOLUTION $\lim_{x\to 5} x^{-2} = 5^{-2} = \frac{1}{25}.$

7. $\lim_{x\to 0.2} (3x+4)$

SOLUTION Using the Sum Law and the Constant Multiple Law:

$$\begin{aligned}\lim_{x\to 0.2}(3x+4) &= \lim_{x\to 0.2} 3x + \lim_{x\to 0.2} 4\\ &= 3\lim_{x\to 0.2} x + \lim_{x\to 0.2} 4 = 3(0.2)+4 = 4.6.\end{aligned}$$

8. $\lim_{x\to \frac{1}{3}} (3x^3+2x^2)$

SOLUTION Using the Sum Law, the Constant Multiple Law and the Powers Law:

$$\begin{aligned}\lim_{x\to\frac{1}{3}}(3x^3+2x^2) &= \lim_{x\to\frac{1}{3}} 3x^3 + \lim_{x\to\frac{1}{3}} 2x^2\\ &= 3\lim_{x\to\frac{1}{3}} x^3 + 2\lim_{x\to\frac{1}{3}} x^2\\ &= 3\left(\frac{1}{3}\right)^3 + 2\left(\frac{1}{3}\right)^2 = \frac{1}{3}.\end{aligned}$$

9. $\lim_{x\to -1} (3x^4-2x^3+4x)$

SOLUTION Using the Sum Law, the Constant Multiple Law and the Powers Law:

$$\begin{aligned}\lim_{x\to -1}(3x^4-2x^3+4x) &= \lim_{x\to -1} 3x^4 - \lim_{x\to -1} 2x^3 + \lim_{x\to -1} 4x\\ &= 3\lim_{x\to -1} x^4 - 2\lim_{x\to -1} x^3 + 4\lim_{x\to -1} x\\ &= 3(-1)^4 - 2(-1)^3 + 4(-1) = 3+2-4 = 1.\end{aligned}$$

10. $\lim_{x\to 8} (3x^{2/3}-16x^{-1})$

SOLUTION Using the Sum Law, the Constant Multiple Law and the Powers Law:

$$\begin{aligned}\lim_{x\to 8}(3x^{2/3}-16x^{-1}) &= \lim_{x\to 8} 3x^{2/3} - \lim_{x\to 8} 16x^{-1}\\ &= 3\lim_{x\to 8} x^{2/3} - 16\lim_{x\to 8} x^{-1}\\ &= 3(8)^{2/3} - 16(8)^{-1} = 3(4) - 2 = 10.\end{aligned}$$

11. $\lim_{x\to 2} (x+1)(3x^2-9)$

SOLUTION Using the Product Law, the Sum Law and the Constant Multiple Law:

$$\begin{aligned}\lim_{x\to 2}(x+1)\left(3x^2-9\right) &= \left(\lim_{x\to 2} x + \lim_{x\to 2} 1\right)\left(\lim_{x\to 2} 3x^2 - \lim_{x\to 2} 9\right)\\ &= (2+1)\left(3\lim_{x\to 2} x^2 - 9\right)\\ &= 3(3(2)^2-9) = 9.\end{aligned}$$

12. $\lim_{x\to \frac{1}{2}} (4x+1)(6x-1)$

SOLUTION Using the Product Law, the Sum Law and the Constant Multiple Law:

$$\begin{aligned}\lim_{x\to 1/2}(4x+1)(6x-1) &= \left(\lim_{x\to 1/2}(4x+1)\right)\left(\lim_{x\to 1/2}(6x-1)\right)\\ &= \left(\lim_{x\to 1/2}4x+\lim_{x\to 1/2}1\right)\left(\lim_{x\to 1/2}6x-\lim_{x\to 1/2}1\right)\\ &= \left(4\lim_{x\to 1/2}x+\lim_{x\to 1/2}1\right)\left(6\lim_{x\to 1/2}x-\lim_{x\to 1/2}1\right)\\ &= \left(4\cdot\frac{1}{2}+1\right)\left(6\cdot\frac{1}{2}-1\right)=3(2)=6.\end{aligned}$$

13. $\lim_{t\to 4}\frac{3t-14}{t+1}$

SOLUTION Using the Quotient Law, the Sum Law and the Constant Multiple Law:

$$\lim_{t\to 4}\frac{3t-14}{t+1}=\frac{\lim_{t\to 4}(3t-14)}{\lim_{t\to 4}(t+1)}=\frac{3\lim_{t\to 4}t-\lim_{t\to 4}14}{\lim_{t\to 4}t+\lim_{t\to 4}1}=\frac{3\cdot 4-14}{4+1}=-\frac{2}{5}.$$

14. $\lim_{z\to 9}\frac{\sqrt{z}}{z-2}$

SOLUTION Using the Quotient Law, the Powers Law and the Sum Law:

$$\lim_{z\to 9}\frac{\sqrt{z}}{z-2}=\frac{\lim_{z\to 9}\sqrt{z}}{\lim_{z\to 9}(z-2)}=\frac{\lim_{z\to 9}\sqrt{z}}{\lim_{z\to 9}z-\lim_{z\to 9}2}=\frac{3}{7}.$$

15. $\lim_{y\to\frac{1}{4}}(16y+1)(2y^{1/2}+1)$

SOLUTION Using the Product Law, the Sum Law, the Constant Multiple Law and the Powers Law:

$$\begin{aligned}\lim_{y\to\frac{1}{4}}(16y+1)(2y^{1/2}+1) &= \left(\lim_{y\to\frac{1}{4}}(16y+1)\right)\left(\lim_{y\to\frac{1}{4}}(2y^{1/2}+1)\right)\\ &= \left(16\lim_{y\to\frac{1}{4}}y+\lim_{y\to\frac{1}{4}}1\right)\left(2\lim_{y\to\frac{1}{4}}y^{1/2}+\lim_{y\to\frac{1}{4}}1\right)\\ &= \left(16\left(\frac{1}{4}\right)+1\right)\left(2\left(\frac{1}{2}\right)+1\right)=10.\end{aligned}$$

16. $\lim_{x\to 2}x(x+1)(x+2)$

SOLUTION Using the Product Law and Sum Law:

$$\begin{aligned}\lim_{x\to 2}x(x+1)(x+2) &= \left(\lim_{x\to 2}x\right)\left(\lim_{x\to 2}(x+1)\right)\left(\lim_{x\to 2}(x+2)\right)\\ &= 2\left(\lim_{x\to 2}x+\lim_{x\to 2}1\right)\left(\lim_{x\to 2}x+\lim_{x\to 2}2\right)\\ &= 2(2+1)(2+2)=24\end{aligned}$$

17. $\lim_{y\to 4}\frac{1}{\sqrt{6y+1}}$

SOLUTION Using the Quotient Law, the Powers Law, the Sum Law and the Constant Multiple Law:

$$\begin{aligned}\lim_{y\to 4}\frac{1}{\sqrt{6y+1}} &= \frac{1}{\lim_{y\to 4}\sqrt{6y+1}}=\frac{1}{\sqrt{6\lim_{y\to 4}y+1}}\\ &= \frac{1}{\sqrt{6(4)+1}}=\frac{1}{5}.\end{aligned}$$

18. $\lim_{w\to 7}\frac{\sqrt{w+2}+1}{\sqrt{w-3}-1}$

SOLUTION Using the Quotient Law, the Sum Law and the Powers Law:

$$\lim_{w\to 7}\frac{\sqrt{w+2}+1}{\sqrt{w-3}-1} = \frac{\lim_{w\to 7}(\sqrt{w+2}+1)}{\lim_{w\to 7}(\sqrt{w-3}-1)} = \frac{\sqrt{\lim_{w\to 7}(w+2)}+1}{\sqrt{\lim_{w\to 7}(w-3)}-1} = \frac{\sqrt{9}+1}{\sqrt{4}-1} = 4.$$

19. $\lim_{x\to -1}\frac{x}{x^3+4x}$

SOLUTION Using the Quotient Law, the Sum Law, the Powers Law and the Constant Multiple Law:

$$\lim_{x\to -1}\frac{x}{x^3+4x} = \frac{\lim_{x\to -1} x}{\lim_{x\to -1} x^3 + 4\lim_{x\to -1} x} = \frac{-1}{(-1)^3+4(-1)} = \frac{1}{5}.$$

20. $\lim_{t\to -1}\frac{t^2+1}{(t^3+2)(t^4+1)}$

SOLUTION Using the Quotient Law, the Product Law, the Sum Law and the Powers Law:

$$\lim_{x\to -1}\frac{t^2+1}{(t^3+2)(t^4+1)} = \frac{\lim_{x\to -1} t^2 + \lim_{x\to -1} 1}{\left(\lim_{x\to -1} t^3 + \lim_{x\to -1} 2\right)\left(\lim_{x\to -1} t^4 + \lim_{x\to -1} 1\right)} = \frac{(-1)^2+1}{((-1)^3+2)((-1)^4+1)} = \frac{2}{(1)(2)} = 1.$$

21. $\lim_{t\to 25}\frac{3\sqrt{t}-\frac{1}{5}t}{(t-20)^2}$

SOLUTION Using the Quotient Law, the Sum Law, the Constant Multiple Law and the Powers Law:

$$\lim_{t\to 25}\frac{3\sqrt{t}-\frac{1}{5}t}{(t-20)^2} = \frac{3\sqrt{\lim_{t\to 25} t}-\frac{1}{5}\lim_{t\to 25} t}{\left(\lim_{t\to 25} t-20\right)^2} = \frac{3(5)-\frac{1}{5}(25)}{5^2} = \frac{2}{5}.$$

22. $\lim_{y\to \frac{1}{3}}(18y^2-4)^4$

SOLUTION Using the Powers Law, the Sum Law and the Constant Multiple Law:

$$\lim_{y\to \frac{1}{3}}(18y^2-4)^4 = \left(18\lim_{y\to \frac{1}{3}} y^2-4\right)^4 = (2-4)^4 = 16.$$

23. $\lim_{t\to \frac{3}{2}}(4t^2+8t-5)^{3/2}$

SOLUTION Using the Powers Law, the Sum Law and the Constant Multiple Law:

$$\lim_{t\to \frac{3}{2}}(4t^2+8t-5)^{3/2} = \left(4\lim_{t\to \frac{3}{2}} t^2+8\lim_{t\to \frac{3}{2}} t-5\right)^{3/2} = (9+12-5)^{3/2} = 64.$$

24. $\lim_{t\to 7}\frac{(t+2)^{1/2}}{(t+1)^{2/3}}$

SOLUTION Using the Quotient Law, the Powers Law and the Sum Law:

$$\lim_{t\to 7}\frac{(t+2)^{1/2}}{(t+1)^{2/3}} = \frac{\left(\lim_{t\to 7} t+2\right)^{1/2}}{\left(\lim_{t\to 7} t+1\right)^{2/3}} = \frac{9^{1/2}}{8^{2/3}} = \frac{3}{4}.$$

25. Use the Quotient Law to prove that if $\lim_{x\to c} f(x)$ exists and is nonzero, then

$$\lim_{x\to c} \frac{1}{f(x)} = \frac{1}{\lim_{x\to c} f(x)}$$

SOLUTION Since $\lim_{x\to c} f(x)$ is nonzero, we can apply the Quotient Law:

$$\lim_{x\to c} \left(\frac{1}{f(x)}\right) = \frac{\left(\lim_{x\to c} 1\right)}{\left(\lim_{x\to c} f(x)\right)} = \frac{1}{\lim_{x\to c} f(x)}.$$

26. Assuming that $\lim_{x\to 6} f(x) = 4$, compute:

(a) $\lim_{x\to 6} f(x)^2$ **(b)** $\lim_{x\to 6} \frac{1}{f(x)}$ **(c)** $\lim_{x\to 6} x\sqrt{f(x)}$

SOLUTION

(a) Using the Powers Law:

$$\lim_{x\to 6} f(x)^2 = \left(\lim_{x\to 6} f(x)\right)^2 = 4^2 = 16.$$

(b) Since $\lim_{x\to 6} f(x) \neq 0$, we may apply the Quotient Law:

$$\lim_{x\to 6} \frac{1}{f(x)} = \frac{1}{\lim_{x\to 6} f(x)} = \frac{1}{4}.$$

(c) Using the Product Law and Powers Law:

$$\lim_{x\to 6} x\sqrt{f(x)} = \left(\lim_{x\to 6} x\right)\left(\lim_{x\to 6} f(x)\right)^{1/2} = 6(4)^{1/2} = 12.$$

In Exercises 27–30, evaluate the limit assuming that $\lim_{x\to -4} f(x) = 3$ *and* $\lim_{x\to -4} g(x) = 1$.

27. $\lim_{x\to -4} f(x)g(x)$

SOLUTION $\lim_{x\to -4} f(x)g(x) = \lim_{x\to -4} f(x) \lim_{x\to -4} g(x) = 3 \cdot 1 = 3.$

28. $\lim_{x\to -4} (2f(x) + 3g(x))$

SOLUTION

$$\begin{aligned}\lim_{x\to -4} (2f(x) + 3g(x)) &= 2\lim_{x\to -4} f(x) + 3\lim_{x\to -4} g(x) \\ &= 2\cdot 3 + 3 \cdot 1 = 6 + 3 = 9.\end{aligned}$$

29. $\lim_{x\to -4} \frac{g(x)}{x^2}$

SOLUTION Since $\lim_{x\to -4} x^2 \neq 0$, we may apply the Quotient Law, then applying the Powers Law:

$$\lim_{x\to -4} \frac{g(x)}{x^2} = \frac{\lim_{x\to -4} g(x)}{\lim_{x\to -4} x^2} = \frac{1}{\left(\lim_{x\to -4} x\right)^2} = \frac{1}{16}.$$

30. $\lim_{x\to -4} \frac{f(x)+1}{3g(x)-9}$

SOLUTION

$$\lim_{x\to -4} \frac{f(x)+1}{3g(x)-9} = \frac{\lim_{x\to -4} f(x) + \lim_{x\to -4} 1}{3\lim_{x\to -4} g(x) - \lim_{x\to -4} 9} = \frac{3+1}{3\cdot 1 - 9} = \frac{4}{-6} = -\frac{2}{3}.$$

31. Can the Quotient Law be applied to evaluate $\lim_{x\to 0} \frac{\sin x}{x}$? Explain.

SOLUTION The limit Quotient Law *cannot* be applied to evaluate $\lim\limits_{x\to 0}\dfrac{\sin x}{x}$ since $\lim\limits_{x\to 0} x = 0$. This violates a condition of the Quotient Law. Accordingly, the rule *cannot* be employed.

32. Show that the Product Law cannot be used to evaluate the limit $\lim\limits_{x\to\pi/2}\left(x-\frac{\pi}{2}\right)\tan x$.

SOLUTION The limit Product Law *cannot* be applied to evaluate $\lim\limits_{x\to\pi/2}(x-\pi/2)\tan x$ since $\lim\limits_{x\to\pi/2}\tan x$ does not exist (for example, as $x\to\pi/2-$, $\tan x\to\infty$). This violates a hypothesis of the Product Law. Accordingly, the rule *cannot* be employed.

33. Give an example where $\lim\limits_{x\to 0}(f(x)+g(x))$ exists but neither $\lim\limits_{x\to 0} f(x)$ nor $\lim\limits_{x\to 0} g(x)$ exists.

SOLUTION Let $f(x)=1/x$ and $g(x)=-1/x$. Then $\lim\limits_{x\to 0}(f(x)+g(x))=\lim\limits_{x\to 0} 0 = 0$ However, $\lim\limits_{x\to 0} f(x)=\lim\limits_{x\to 0} 1/x$ and $\lim\limits_{x\to 0} g(x)=\lim\limits_{x\to 0} -1/x$ do not exist.

Further Insights and Challenges

34. Show that if both $\lim\limits_{x\to c} f(x)\,g(x)$ and $\lim\limits_{x\to c} g(x)$ exist and $\lim\limits_{x\to c} g(x)\neq 0$, then $\lim\limits_{x\to c} f(x)$ exists. *Hint:* Write $f(x)=\dfrac{f(x)\,g(x)}{g(x)}$.

SOLUTION Given that $\lim\limits_{x\to c} f(x)g(x)=L$ and $\lim\limits_{x\to c} g(x)=M\neq 0$ both exist, observe that

$$\lim_{x\to c} f(x)=\lim_{x\to c}\frac{f(x)g(x)}{g(x)}=\frac{\lim\limits_{x\to c} f(x)g(x)}{\lim\limits_{x\to c} g(x)}=\frac{L}{M}$$

also exists.

35. Suppose that $\lim\limits_{t\to 3} t\,g(t)=12$. Show that $\lim\limits_{t\to 3} g(t)$ exists and equals 4.

SOLUTION We are given that $\lim\limits_{t\to 3} t\,g(t)=12$. Since $\lim\limits_{t\to 3} t = 3\neq 0$, we may apply the Quotient Law:

$$\lim_{t\to 3} g(t)=\lim_{t\to 3}\frac{t\,g(t)}{t}=\frac{\lim\limits_{t\to 3} t\,g(t)}{\lim\limits_{t\to 3} t}=\frac{12}{3}=4.$$

36. Prove that if $\lim\limits_{t\to 3}\frac{h(t)}{t}=5$, then $\lim\limits_{t\to 3} h(t)=15$.

SOLUTION Given that $\lim\limits_{t\to 3}\dfrac{h(t)}{t}=5$, observe that $\lim\limits_{t\to 3} t = 3$. Now use the Product Law:

$$\lim_{t\to 3} h(t)=\lim_{t\to 3} t\,\frac{h(t)}{t}=\left(\lim_{t\to 3} t\right)\left(\lim_{t\to 3}\frac{h(t)}{t}\right)=3\cdot 5=15.$$

37. Assuming that $\lim\limits_{x\to 0}\frac{f(x)}{x}=1$, which of the following statements is necessarily true? Why?

(a) $f(0)=0$

(b) $\lim\limits_{x\to 0} f(x)=0$

SOLUTION

(a) Given that $\lim\limits_{x\to 0}\dfrac{f(x)}{x}=1$, it is not necessarily true that $f(0)=0$. A counterexample is provided by $f(x)=\begin{cases} x, & x\neq 0\\ 5, & x=0\end{cases}$.

(b) Given that $\lim\limits_{x\to 0}\dfrac{f(x)}{x}=1$, it is necessarily true that $\lim\limits_{x\to 0} f(x)=0$. For note that $\lim\limits_{x\to 0} x = 0$, whence

$$\lim_{x\to 0} f(x)=\lim_{x\to 0} x\,\frac{f(x)}{x}=\left(\lim_{x\to 0} x\right)\left(\lim_{x\to 0}\frac{f(x)}{x}\right)=0\cdot 1=0.$$

38. Prove that if $\lim\limits_{x\to c} f(x)=L\neq 0$ and $\lim\limits_{x\to c} g(x)=0$, then the limit $\lim\limits_{x\to c}\frac{f(x)}{g(x)}$ does not exist.

SOLUTION Suppose that $\lim\limits_{x\to c}\dfrac{f(x)}{g(x)}$ exists. Then

$$L=\lim_{x\to c} f(x)=\lim_{x\to c} g(x)\cdot\frac{f(x)}{g(x)}=\lim_{x\to c} g(x)\cdot\lim_{x\to c}\frac{f(x)}{g(x)}=0\cdot\lim_{x\to c}\frac{f(x)}{g(x)}=0.$$

But, we were given that $L\neq 0$, so we have arrived at a contradiction. Thus, $\lim\limits_{x\to c}\dfrac{f(x)}{g(x)}$ does not exist.

39. Suppose that $\lim_{h\to 0} g(h) = L$.

(a) Explain why $\lim_{h\to 0} g(ah) = L$ for any constant $a \neq 0$.

(b) If we assume instead that $\lim_{h\to 1} g(h) = L$, is it still necessarily true that $\lim_{h\to 1} g(ah) = L$?

(c) Illustrate (a) and (b) with the function $f(x) = x^2$.

SOLUTION

(a) As $h \to 0$, $ah \to 0$ as well; hence, if we make the change of variable $w = ah$, then

$$\lim_{h\to 0} g(ah) = \lim_{w\to 0} g(w) = L.$$

(b) No. As $h \to 1$, $ah \to a$, so we should not expect $\lim_{h\to 1} g(ah) = \lim_{h\to 1} g(h)$.

(c) Let $g(x) = x^2$. Then

$$\lim_{h\to 0} g(h) = 0 \quad \text{and} \quad \lim_{h\to 0} g(ah) = \lim_{h\to 0} (ah)^2 = 0.$$

On the other hand,

$$\lim_{h\to 1} g(h) = 1 \quad \text{while} \quad \lim_{h\to 1} g(ah) = \lim_{h\to 1} (ah)^2 = a^2,$$

which is equal to the previous limit if and only if $a = \pm 1$.

40. Assume that $L(a) = \lim_{x\to 0} \dfrac{a^x - 1}{x}$ exists for all $a > 0$. Assume also that $\lim_{x\to 0} a^x = 1$.

(a) Prove that $L(ab) = L(a) + L(b)$ for $a, b > 0$. *Hint:* $(ab)^x - 1 = a^x(b^x - 1) + (a^x - 1)$. This shows that $L(a)$ "behaves" like a logarithm. We will see that $L(a) = \ln a$ in Section 3.10.

(b) Verify numerically that $L(12) = L(3) + L(4)$.

SOLUTION

(a) Let $a, b > 0$. Then

$$\begin{aligned} L(ab) = \lim_{x\to 0} \frac{(ab)^x - 1}{x} &= \lim_{x\to 0} \frac{a^x(b^x - 1) + (a^x - 1)}{x} \\ &= \lim_{x\to 0} a^x \cdot \lim_{x\to 0} \frac{b^x - 1}{x} + \lim_{x\to 0} \frac{a^x - 1}{x} \\ &= 1 \cdot L(b) + L(a) = L(a) + L(b). \end{aligned}$$

(b) From the table below, we estimate that, to three decimal places, $L(3) = 1.099$, $L(4) = 1.386$ and $L(12) = 2.485$. Thus,

$$L(12) = 2.485 = 1.099 + 1.386 = L(3) + L(4).$$

x	-0.01	-0.001	-0.0001	0.0001	0.001	0.01
$(3^x - 1)/x$	1.092600	1.098009	1.098552	1.098673	1.099216	1.104669
$(4^x - 1)/x$	1.376730	1.385334	1.386198	1.386390	1.387256	1.395948
$(12^x - 1)/x$	2.454287	2.481822	2.484600	2.485215	2.488000	2.516038

2.4 Limits and Continuity

Preliminary Questions

1. Which property of $f(x) = x^3$ allows us to conclude that $\lim_{x\to 2} x^3 = 8$?

SOLUTION We can conclude that $\lim_{x\to 2} x^3 = 8$ because the function x^3 is continuous at $x = 2$.

2. What can be said about $f(3)$ if f is continuous and $\lim_{x\to 3} f(x) = \frac{1}{2}$?

SOLUTION If f is continuous and $\lim_{x\to 3} f(x) = \frac{1}{2}$, then $f(3) = \frac{1}{2}$.