Section 1: True or False (Worth 5 points each)
Directions: For each statement below, write True if the statement MUST BE TRUE! Write False if it is false or MAY BE TRUE.

_____ #1. If \( f'(a) = 0 \) and \( f''(a) = 0 \), then \( f(x) \) has a relative maximum at \( x = a \).

_____ #2. If \( f'(x) \) is increasing at \( x = b \), then \( f(x) \) is concave up at \( x = b \).

_____ #3. An inflection point occurs where \( f(x) \) has a relative maximum.

_____ #4. \( f(x) \) is decreasing at \( x = c \) if \( f'(x) < 0 \) at \( x = c \).

_____ #5. If \( f'(x) \) has a relative minimum at a given point, then \( f(x) \) has a point of inflection at that point.

Section 2: Multiple-Choice (Worth 5 points each)
Directions: For each problem below, choose the ONE BEST answer.
Please circle your answer.

#6.

The equation of the tangent to \( y = (x - 1)^5 \) at its \( y \) intercept is: (3 marks)

a) \( y = x + 1 \) b) \( y = 0 \) c) \( y = 5x - 1 \) d) \( y = -5x - 1 \) e) \( y = x - 1 \)

#7.

Given that \( f'(x) = (x - 1)(2x - 4)(x - 4)^2 \), which of the following statement(s) are/is true?

I) \( f(x) \) reaches a relative maximum at \( x = 1 \)

II) \( f(x) \) is decreasing at \( x = 3 \)

III) \( f(x) \) is concave up at \( x = 5 \)

a) I only b) II only

c) III only d) I and III

c) I, II, and III
#8.
A ladder which is 5 meters long slips down a wall at a rate of 2 meters per second. How fast, in meters per second, is the base of the ladder moving away from the wall at the instant when its height is above the ground 3 meters?

a) 1  
b) 1.5  
c) 2  
d) 2.5  
e) 3

#9.
The position of a particle is given by the formula \( x(t) = t^3 - 4t^2 + 10 \). At time 2 seconds, which of the following statements is correct?

a) The particle's velocity is increasing  
b) The particle's velocity is decreasing  
c) The particle is moving towards zero  
d) The particle is at rest

#10.
A cone has a radius of 5 cm and a height of 15 cm. It is empty and is being filled at a rate of 12\(\pi\) cubic cm per second. Find the rate of change of change of the radius in cm per second when the radius of the water is 2 cm.

a) 0.5  
b) 1  
c) 1.5  
d) 2.5  
e) 3

#11.
The management of a large store wishes to add a fenced-in rectangular storage yard of 20,000 square feet, using the building as one side of the yard. Find the minimum amount of fencing that must be used to enclose the remaining 3 sides of the yard.

(a) 400 ft  
(b) 200 ft  
(c) 20,000 ft  
(d) 500 ft  
(e) None of these

#12.
The figure given is the graph of the second derivative of a polynomial function, \( f \). Choose a graph of \( f \).

\[ f''(x) \]

\[ f''(x) \]

\[ f''(x) \]

\[ f''(x) \]

\[ f''(x) \]
#13.

Suppose \( f \) is a continuous and differentiable function of \( x \) on the intervals \([-4, 0)\) and \((0, 1]\).

Suppose also that we have the following table of data for the function:

\[
\begin{array}{c|c}
  x & f(x) \\
  \hline
  -4 & .5 \\
  -2 & .75 \\
  .5 & 2 \\
  1 & .75 \\
\end{array}
\]

Which of the following statements is NOT necessarily true?

A. There exists a \( c \) between \(-2\) and \(1\) for which \( f'(c) = -2.5 \).

B. There exists a \( c \) between \(.5\) and \(1\) for which \( f'(c) = .75 \).

C. There exists a \( c \) between \(-4\) and \(0\) for which \( f''(c) = .125 \).

D. There exists a \( c \) between \(-4\) and \(-2\) such that \( f(c) = .7 \).

E. There exists a \( c \) between \(.5\) and \(1\) such that \( f(c) = 1 \).

Section 3: Free Response (Worth 7 points each)
Directions: Show all work for credit.

#14. (attach work on separate paper)
Determine whether Rolle’s Theorem can be applied to \( f(x) = \cos x \) on the interval \([0, 2\pi]\). If Rolle’s Theorem can be applied, find all values that satisfy the theorem. If Rolle’s Theorem cannot be applied, state why not.

#15. (attach sketch on separate paper)
Sketch a graph meeting the following criteria.
\( f(x) \) is decreasing on the intervals \((-\infty, -2) \) and \((0, 2) \).
\( f(x) \) is increasing on the intervals \((-2, 0) \) and \((2, \infty) \).
\( f(x) \) is concave downward on the intervals \((-\infty, -2), (-2, 2), \) and \((2, \infty) \).
\( f(x) \) is not differentiable at \( x = -2 \) and \( x = 2 \).
\( f(-2) = -1 \)
\( f(0) = 3 \)
\( f(2) = -3 \)
Section 4: Fill-In (Worth 1 point per cell)

#16.

For the graph of \( f(x) \) shown, fill in the table below with + (positive), - (negative), or 0.

<table>
<thead>
<tr>
<th>Point</th>
<th>( f(x) )</th>
<th>( f'(x) )</th>
<th>( f''(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
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App Der Test (by 2008-2009)

1) False \[\Rightarrow\] $y$ counter example.
2) True \[\Rightarrow\] $f^\prime\prime(x)$ inc. $\Rightarrow f^\prime\prime(x) > 0 \Rightarrow f(x)$ con. up.
3) False \[\Rightarrow\] Pol when $f^\prime\prime(x)$ has extrema, not $f(x)$
4) True \[\Rightarrow\] def of dec function
5) True \[\Rightarrow\] $f^\prime(x)$ rel min $\Rightarrow f^\prime(x)$ changes from $- \to + \Rightarrow$ Pol

(e) $y = (x-1)^5$
$y' = 5(x-1)^4$
$y\mid_{x=0} \Rightarrow x = 0$

$m_{tan}\mid_{x=0} \Rightarrow$

$5(0-1)^4 = 5(-1)^4$
$5(1) = 5$

$y_{+1} = 5(x-0)$
$y = 5x-1$

7) $f'(x) = (x-1)(2x-4)(x-4)^2$
$f'(1) = 0$
$f'(3) = (+)(+)(1) = +$ increasing

$max_{x=1/2}$

$f'(1/2) = (-)(-)(1)$
$f'(3/2) = (+)(-)(1)$

$\mu \mid_{1/2}$

$\mu \mid_{3/2}$
\[
\begin{align*}
\frac{dh}{dt} &= \frac{r}{t} \\
\frac{dr}{dt} &= \frac{h}{r} \\
\frac{dh}{dx} &= \frac{h}{r} + \frac{r}{h} \\
\frac{dr}{dx} &= \frac{h}{r} + \frac{r}{h} \\
x^2 + y^2 &= 25 \\
2x \frac{dx}{dx} + 2y \frac{dy}{dx} &= 0 \\
\frac{dy}{dx} &= -\frac{x}{y} \\
h &= 15 \\
y &= 20 \\
x &= 3 \\
\int f''(x) dx &= \int (x^2 - 6x + 10)(x - y) \frac{dy}{dx}^- \\
&= \left(\frac{x^3}{3} - 3x^2 + 5x + C\right) \\
f(x) &= x^3 - 6x^2 + 5x + C \\
f(3) &= 27 - 54 + 15 + C \\
&= -6 + C
\end{align*}
\]
\( a(t) = 6t - 8 \)  \( \therefore t = 2 \)

\( a(2) = 12 - 8 = + \checkmark \)

10) \[ V = \frac{2}{3} \pi r^2 h \]
\[ V = \frac{2}{3} \pi r^3 \]
\[ \frac{dV}{dt} = \pi r^2 \frac{dr}{dt} \]
\[ \frac{5}{r} = \frac{15}{h} \]
\[ 5h = 15r \]
\[ h = 3r \]

Find \( \frac{dr}{dt} \) when \( r = 2 \)

\[ 12 \pi = 3 \pi (4) \frac{dr}{dt} \]
\[ 12 \pi = 12 \pi \frac{dr}{dt} \]
\[ l = \frac{dr}{dt} \]
1) \[ xy = 20000 \]
\[ y = \frac{20,000}{x} \]
\[ P = 2x + y \]
\[ P = 2x + 20,000 \times^{-1} \]
\[ P'(x) = 2 - \frac{20,000}{x^2} \]

\[ 2 - \frac{20,000}{x^2} = 0 \Rightarrow 2 = \frac{20,000}{x^2} \]
\[ 2x^2 = 20,000 \]
\[ x^2 = 10,000 \]
\[ x = 100 \]

Verify \[ P''(x) = \frac{40,000}{x^3} \]
\[ P''(100) = \frac{40,000}{(100)^3} = + \uparrow \uparrow \min \]
\[ P = 2x + y = 2(100) + 200 \]
\[ y = \frac{20,000}{100} \]

\[ 400 \text{ ft} \]
12) \( \begin{align*}
\text{Test a)} \\
(0.5, 2), (1, 1.75) \\
m &= \frac{1.75 - 2}{1 - 0.5} = -2.5 \text{ (true)}
\end{align*} \)

13) \( \begin{align*}
\text{Test b)} & \text{ False by process of elimination} \\
\text{Test c)} (-4, 0.5) (-2, 1.75) \\
m &= \frac{1.75 - 0.5}{-2 + 4} = 0.75 \text{ (false)}
\end{align*} \)

\( \begin{align*}
\text{Test e)} \\
& f(0.5) = 2 \quad f(1) = 1.75 \quad 0.75 < 1 < 2 \text{ (true)}
\end{align*} \)

14) \( f(x) = \cos x \) - cont & cliff

\( f(a) = f(b) ? \)

\( f(0) = \cos(0) = 1 \)

\( f(2\pi) = \cos(2\pi) = 1 \)

\( f''(x) = -\sin x = 0 \)

\( \sin x = 0 \)

\( @ \ x = 0, \pi, 2\pi \text{ (true)} \in [0, 2\pi] \)
(a) Answers will vary

(b) | $f(x)$ | $f'(x)$ | $f''(x)$ |
<table>
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</tbody>
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$\text{or } 0 + \text{ or } 0 \text{ will appear}$