Let $R$ be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the $y$-axis, as shown in the figure above.

(a) Find the area of the region $R$.

(b) Find the volume of the solid generated when the region $R$ is revolved about the $x$-axis.

(c) The region $R$ is the base of a solid. For this solid, each cross section perpendicular to the $x$-axis is a square. Find the volume of this solid.

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**Region $R$**

$e^{-x^2} = 1 - \cos x$ at $x = 0.941944 = A$

(a) Area $= \int_0^A (e^{-x^2} - (1 - \cos x))\,dx$

$= 0.590$ or $0.591$

(b) Volume $= \pi \int_0^A \left( (e^{-x^2})^2 - (1 - \cos x)^2 \right)\,dx$

$= 0.55596\pi = 1.746$ or $1.747$

(c) Volume $= \int_0^A \left( e^{-x^2} - (1 - \cos x) \right)^2 \,dx$

$= 0.461$