Coordinate Algebra Agenda

Go over homework (#’s 21-30 on EOCT review)
Begin section 9.3 (Residual plots)
HW: #’s 31-40 on EOCT review and we’ll see how far we get in the book.
22 more class days until the EOCT!!!
Review
21.) Caitlyn
22.) Joseph
23.) Ben
24.) Louis
25.) Eddie
26.) Brett
27.) Mallory
28.) Kayla
29.) Parker
30.) Madison
Determine whether each graph represents a function with an absolute minimum, an absolute maximum, or neither.

21. \( \checkmark \) Absolute minimum

22. \( \checkmark \) Neither

23. \( \checkmark \) Absolute maximum
Create an equation and sketch a graph for a function with each set of given characteristics. Use values that are any real numbers between –10 and 10.

24. Create an equation and sketch a graph that:
• is linear,
• is discrete, and
• is decreasing across the entire domain.

\[ y = mx + b \]
\[ y = -1x \]
\[ y = -x \]
25. Create an equation and sketch a graph that:
- is a smooth curve,
- is increasing across the entire domain,
- is continuous, and
- is exponential.

\[
y = 2^x
\]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>-1</td>
<td>\frac{1}{2}</td>
</tr>
</tbody>
</table>
Use each scenario to complete the table of values and calculate the unit rate of change.

26. The football boosters sell hooded sweatshirts to raise money for new equipment. Each sweatshirt costs $18.

<table>
<thead>
<tr>
<th>Independent Quantity</th>
<th>Dependent Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity</td>
<td></td>
</tr>
<tr>
<td>Units</td>
<td></td>
</tr>
<tr>
<td>Expression</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th># of Shirts</th>
<th>Money</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>90</td>
</tr>
<tr>
<td>10</td>
<td>180</td>
</tr>
<tr>
<td>20</td>
<td>360</td>
</tr>
<tr>
<td>30</td>
<td>540</td>
</tr>
<tr>
<td>40</td>
<td>720</td>
</tr>
</tbody>
</table>
Solve each function for the given input value. The function $A(t) = 7t$ represents the total amount of money in dollars Carmen earns babysitting as a function of time in hours.

27. $A(3) = 21 \Rightarrow 7(3)$

28. $A(4.5) = 31.5 \Rightarrow 7(4.5)$
Use the graph to determine the input value for each given output value. The function $D(t) = 40t$ represents the total distance traveled in miles as a function of time in hours.

$D(t) = 40 + t$

$120 = 40t$

$3 = t$

29. $D(t) = 120$

30. $D(t) = 320$
The Residual Effect
Creating Residual Plots

LEARNING GOALS
In this lesson, you will:
- Create residual plots.
- Analyze the shapes of residual plots.

KEY TERMS
- residual
- residual plot

Maybe you once made a lot of spelling mistakes in an essay that you wrote. The next time you wrote an essay, you made sure to do a spell check (or use a dictionary). Maybe you noticed that you missed a lot of free throws in basketball games. You decided to practice your free throw shooting to improve. Maybe you told a joke that hurt your friend’s feelings. You remembered to be more sensitive around him or her in the future.

We all learn from our mistakes. In mathematics, too, you can learn a lot about data by looking at error. That’s what this lesson is all about!
You have used the shape of data in a scatter plot and the correlation coefficient to help you determine whether a linear model is an appropriate model for a data set. For some data sets, these measures may not provide enough information to determine if a linear model is most appropriate.

In order to be a safe driver, there are a lot of things to consider. For example, you have to leave enough distance between your car and the car in front of you in case you need to stop suddenly. The table shows the braking distance for a particular car when traveling at different speeds.

<table>
<thead>
<tr>
<th>Speed (mph)</th>
<th>Braking Distance (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>48</td>
</tr>
<tr>
<td>40</td>
<td>80</td>
</tr>
<tr>
<td>50</td>
<td>120</td>
</tr>
<tr>
<td>60</td>
<td>180</td>
</tr>
<tr>
<td>70</td>
<td>240</td>
</tr>
<tr>
<td>80</td>
<td>320</td>
</tr>
</tbody>
</table>

1. Construct a scatter plot of the data.
1. Construct a scatter plot of the data.

Diagram:
- Y-axis labeled as "Speed (mph)
- X-axis labeled as "Braking Distance (d)"
2. Based on the shape of the scatter plot, do you think a linear model is appropriate? Explain your reasoning.

Yes. It looks like they line up.

3. Calculate the line of best fit for the data. Write a function $d(s)$ to represent the line of best fit.

$y = 5.42x - 133.9$

$d(s) = 5.42s - 133.9$

4. Interpret the function in terms of the problem situation.

Breaking inc. 5.42 ft per 1 mph.

Distance

5. Determine and interpret the correlation coefficient.

$r = 0.98$

Strong positive correlation.

6. Complete the table to determine the residuals for the braking distance data.

<table>
<thead>
<tr>
<th>Speed (mph)</th>
<th>Observed Braking Distance (feet)</th>
<th>Predicted Braking Distance (feet)</th>
<th>Residual Value Observed Value – Predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>48</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>120</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>180</td>
<td></td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>240</td>
<td></td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>320</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In addition to the shape of the scatter plot and the correlation coefficient, one additional method to determine if a linear model is appropriate for the data is to analyze the residuals. A **residual** is the distance between an observed data value and its predicted value using the regression equation.

6. Complete the table to determine the residuals for the braking distance data.

\[
y = 5.42x - 133.9
\]

<table>
<thead>
<tr>
<th>Speed (mph)</th>
<th>Observed Braking Distance (feet)</th>
<th>Predicted Braking Distance (feet)</th>
<th>Residual Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>48</td>
<td>28.7</td>
<td>19.3</td>
</tr>
<tr>
<td>40</td>
<td>80</td>
<td>82.9</td>
<td>-2.9</td>
</tr>
<tr>
<td>50</td>
<td>120</td>
<td>137.1</td>
<td>-17.1</td>
</tr>
<tr>
<td>60</td>
<td>180</td>
<td>191.3</td>
<td>-11.3</td>
</tr>
<tr>
<td>70</td>
<td>240</td>
<td>245.5</td>
<td>-5.5</td>
</tr>
<tr>
<td>80</td>
<td>320</td>
<td>299.7</td>
<td>+20.3</td>
</tr>
</tbody>
</table>
Now, let's analyze the relationship between the observed braking distances and the predicted braking distances using graphs. The graph of the line of best fit for the observed braking distances is shown. Use the graph to answer Questions 7–9 and then construct a residual plot.

7. For each data point, there is a residual equal to the difference between the observed measured braking distance and the value predicted by the line of best fit.
   a. Plot each observed value on the Braking Distance graph.
   b. Connect each observed value to its predicted value using a vertical line.
7. For each data point, there is a residual equal to the difference between the observed measured braking distance and the value predicted by the line of best fit.
   a. Plot each observed value on the Braking Distance graph.
   b. Connect each observed value to its predicted value using a vertical line.

The vertical distance from each observed data point to the line is called the residual for that x-value.
8. Examine the scatter plot and the residual values.
   a. When does a residual have a positive value?
      when the observed value is greater than the predicted value.
   b. When does a residual have a negative value?
      when the predicted value is greater than the observed value.

The residual data can now be used to create a residual plot.
A residual plot is a scatter plot of the independent variable on the x-axis and the residuals on the y-axis.

9. Construct a residual plot of the speed and braking distance data.

10. Interpret each residual in the context of the problem situation.
    - At 30 mph, the braking distance is 20 feet greater than predicted.
    - At 40 mph, the braking distance is ____________________________.
    - At 50 mph, the braking distance is ____________________________.
    - At 60 mph, the braking distance is ____________________________.
    - At 70 mph, the braking distance is ____________________________.
    - At 80 mph, the braking distance is ____________________________.

11. What pattern, if any, do you notice in the residuals?
10. Interpret each residual in the context of the problem situation.
   - At 30 mph, the braking distance is 16 feet greater than predicted.
   - At 40 mph, the braking distance is 19.9 less than predicted.
   - At 50 mph, the braking distance is 17.1 < predicted.
   - At 60 mph, the braking distance is 11.3 < predicted.
   - At 70 mph, the braking distance is 5.5 < predicted.
   - At 80 mph, the braking distance is 20.3 > predicted.

11. What pattern, if any, do you notice in the residuals?
   Predicted value is farther from actual value at the ends of the data range.
The shape of the residual plot can be useful to determine whether there may be a more appropriate model other than a linear model for a data set.

If a residual plot results in no identifiable pattern or a flat pattern, then the data may be linearly related. If there is a pattern in the residual plot, the data may not be linearly related. Even if the data are not linearly related, the data may still have some other type of non-linear relationship.

Residual Plots Indicating a Non-Linear Relationship

There is a pattern in the residual plot. As the x-value increases, the residuals become more spread out. The data may not be linearly related.

There is a pattern in the residual plot. The residuals form a curved pattern. The data may not be linearly related.

12. Interpret the residual plot for the braking distance data.
Residual Plots

There is no pattern in the residual plot. The data may be linearly related.

There is a flat pattern in the residual plot. The data may be linearly related.

Residual Plots Indicating a Non-Linear Relationship

There is a pattern in the residual plot. As the x-value increases, the residuals become more spread out. The data may not be linearly related.

There is a pattern in the residual plot. The residuals form a curved pattern. The data may not be linearly related.

12. Interpret the residual plot for the braking distance data.
1. Explain what you can conclude from each residual plot about whether a linear model is appropriate.

a. 

b. 

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2. How would you describe the difference between “line of best fit” and “most appropriate model”?

AND 31-40 EOCT REV.